There Must Be a Rational Explanation

8.1

Adding and Subtracting Rational Expressions

LEARNING GOALS

In this lesson, you will:

- Add and subtract rational expressions.
- Factor to determine a least common denominator.

At some point in most people's lives, the task of putting together a jigsaw puzzle is just one of the things everyone seems to do. In a jigsaw puzzle, small, interlocking pieces with colors or designs fit together to make a larger picture. The pieces can only fit together one way, so various strategies are employed to determine which pieces fit together. Younger children are often encouraged to work on jigsaw puzzles as a way of developing problem-solving skills. Jigsaw puzzles are popular among adults, too, and some of them can get quite complicated. Sometimes complex designs, a large number of pieces, or even 3-dimensional platforms can make puzzles very challenging. A lot of time, skill, and patience are required to put these puzzles together. One particular puzzle has 24,000 pieces to it! That certainly isn't child's play!

People who are serious about puzzles can qualify to be on a national puzzling team and compete in international competitions. The most prestigious competition is held every year in Belgium. What types of puzzles or board games do you like? Do you know anybody who is really good at putting puzzles together?

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PROBLEM 1 Oh Snap . . . Look at the Denominator on that Rational

Previously, you learned that dividing polynomials was just like dividing integers. Well, performing operations on rational expressions involving variables is just like performing operations on rational numbers.

	Rational Numbers	Rational Expressions Involving Variables in the Numerator
Example 1	$\frac{1}{6}+\frac{5}{6}-\frac{1}{6}=\frac{5}{6}$	$\frac{1x}{6}+\frac{5x}{6}-\frac{1x}{6}=\frac{5x}{6}$
Example 2	$\frac{3}{2}+\frac{2}{5}-\frac{3}{4}=\frac{3(10)}{2(10)}+\frac{2(4)}{5(4)}-\frac{3(5)}{4(5)}$ $=\frac{30}{20}+\frac{8}{20}-\frac{15}{20}$ $=\frac{23}{20}$	$\frac{3x}{2}+\frac{2y}{5}-\frac{3x}{4}=\frac{3(10)x}{2(10)}+\frac{2(4)y}{5(4)}-\frac{3(5)x}{4(5)}$ $=\frac{30x}{20}+\frac{8y}{20}-\frac{15x}{20}$ $=\frac{15x+8y}{20}$
Example 3	$\frac{3}{5}+\frac{2}{3}-\frac{2}{15}=\frac{3(3)}{5(3)}+\frac{2(5)}{3(5)}-\frac{2}{15}$ $=\frac{9}{15}+\frac{10}{15}-\frac{2}{15}$ $=\frac{17}{15}$	$\frac{3x}{5}+\frac{2y}{3}-\frac{2}{15}-\frac{2x+3y}{5}=\frac{3(3)x}{5(3)}+\frac{2(5)y}{3(5)}-\frac{2}{15}-\frac{(2x+3y)(3)}{5(3)}$ $=\frac{9x}{15}+\frac{10y}{15}-\frac{2}{15}-\frac{6x+9y}{15}$ $=\frac{9x+10y-2-(6x+9y)}{15}$ $=\frac{9x+10y-2-6x-9y}{15}$ $=\frac{3x+y-2}{15}$

1. Analyze the examples.

   a. Explain the process used to add and subtract each expression.

   b. In Example 2, why is $\frac{3}{2}=\frac{3(10)}{2(10)}$ and why is $\frac{3x}{2}=\frac{3(10)x}{2(10)}$?

   c. In Example 3, explain how $-\frac{(2x+3y)(3)}{5(3)}=\frac{-6x-9y}{15}$.

2. Analyze Noelle's work.

Noelle

$$\frac{3x}{3} + \frac{2x}{8} - \frac{1}{2}$$

To determine a common denominator, I multiply all the denominators together: $3 \cdot 8 \cdot 2 = 48$

$$\frac{3x(16)}{3(16)} + \frac{2x(6)}{8(6)} - \frac{1(24)}{2(24)} = \frac{48x}{48} + \frac{12x}{48} - \frac{24}{48}$$

$$= \frac{60x - 24}{48}$$

$$= \frac{5x - 2}{4}$$

Explain how Noelle could have added the rational expressions more efficiently.

3. Calculate each sum and difference.

a. $\frac{3}{6} + \frac{5x}{4} - \frac{y}{8}$

b. $\frac{x - 2y}{3} + \frac{x}{12} - \frac{z}{4}$

c. $\frac{4x}{6} - \frac{2x}{9} - \frac{x}{18}$

4. Is the set of rational expressions closed under addition and subtraction? Explain your reasoning.

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5. Notice that all the variables in the right column of the table are in the numerator. If there were variables in the denominator, do you think the process of adding and subtracting the expression would change? Explain your reasoning

PROBLEM 2 Umm, I Think There Are Some Variables in Your Denominator . . .

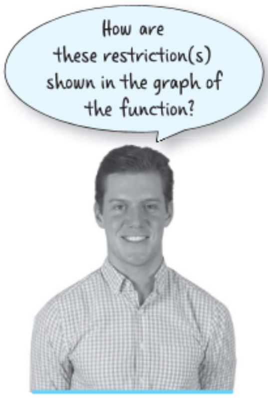
When rational expressions contain variables in the denominator, the process remains the same—you still need to determine the least common denominator (LCD) before adding and subtracting.

It will save time and effort if you determine the LCD and use it to add and subtract rational expressions.

1. Consider Method A compared to Method B in both columns of the table.

	Rational Numbers	Rational Expressions Involving Variables in the Denominator
Method A	$\begin{aligned}\frac{1}{3}+\frac{1}{3^2}&=\frac{1(3^2)}{3(3^2)}+\frac{1(3)}{3^2(3)}\\&=\frac{3^2}{3^3}+\frac{3}{3^3}\\&=\frac{3^2+3}{3^3}\\&=\frac{3(3+1)}{3(3^2)}\\&=\frac{3+1}{3^2}\end{aligned}$	$\begin{aligned}\frac{1}{x}+\frac{1}{x^2}&=\frac{1(x^2)}{x(x^2)}+\frac{1(x)}{x^2(x)}\\&=\frac{x^2}{x^3}+\frac{x}{x^3}\\&=\frac{x^2+x}{x^3}\\&=\frac{x(x+1)}{x(x^2)}\\&=\frac{x+1}{x^2}\\\frac{1}{x}+\frac{1}{x^2}&=\frac{x+1}{x^2}\text{ for } x\neq 0\end{aligned}$
Method B	$\frac{1}{3}+\frac{1}{3^2}=\frac{1(3)}{3(3)}+\frac{1}{3^2}=\frac{4}{3^2}$	$\begin{aligned}\frac{1}{x}+\frac{1}{x^2}&=\frac{1(x)}{x(x)}+\frac{1}{x^2}\\&=\frac{x+1}{x^2}\\\frac{1}{x}+\frac{1}{x^2}&=\frac{x+1}{x^2}\text{ for } x\neq 0\end{aligned}$

a. Explain the difference in the methods.



b. Explain why the statement $\frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$ has the restriction $x \neq 0$?

2. Ruth and Samir determine the LCD for the expression: $\frac{1}{x^2 - 1} + \frac{1}{x + 1}$.

Ruth

$\frac{1}{x^2 - 1} + \frac{1}{x + 1}$

$(x^2 - 1)(x + 1)$

LCD: $x^3 + x^2 - x - 1$

Samir

$\frac{1}{x^2 - 1} + \frac{1}{x + 1}$

$\frac{1}{(x - 1)(x + 1)} + \frac{1}{x + 1}$

$(x - 1)(x + 1)$

LCD: $x^2 - 1$

a. Who is correct? Explain your reasoning.

b. Describe any restriction(s) for the value of x.

3. Calculate the least common denominator for each rational expression.

a. $\frac{3}{x + 4}, \frac{7x}{x - 4}$

LCD:

b. $\frac{-2}{3x - 2}, \frac{4x}{3x^2 + 7x - 6}$

LCD:

c. $\frac{-11}{x}, \frac{7}{x - 4}, \frac{x}{x^2 - 16}$

LCD:

d. $\frac{2x}{x^2 - 5x + 6}, \frac{7x + 11}{x^2 - 6x + 9}$

LCD:

Notice that even though there are binomials in the denominator, operations are performed the same as with the set of rational numbers.

Rational Expressions Involving Binomials in the Denominator

$$\frac{1}{x^2 - 1} - \frac{1}{x + 1} = \frac{1}{(x + 1)(x - 1)} - \frac{1}{x + 1}$$

$$= \frac{1}{(x + 1)(x - 1)} - \frac{1(x - 1)}{(x + 1)(x - 1)}$$

$$= \frac{1}{(x + 1)(x - 1)} - \frac{x - 1}{(x + 1)(x - 1)}$$

$$= \frac{1 - x + 1}{(x + 1)(x - 1)}$$

$$= \frac{-x + 2}{(x + 1)(x - 1)}$$

4. Marissa and Salvatore add $\frac{2x+2}{x+1} + \frac{1}{x}$.

Marissa

$$\frac{2x+2}{x+1} + \frac{1}{x} = \frac{(2x+2)(x)}{(x+1)(x)} + \frac{1(x+1)}{x(x+1)}$$
$$= \frac{2x^2+2x}{(x+1)(x)} + \frac{x+1}{x(x+1)}$$
$$= \frac{2x^2+2x+x+1}{x(x+1)}$$
$$= \frac{2x^2+3x+1}{x(x+1)}$$
$$= \frac{(2x+1)\cancel{(x+1)}}{x\cancel{(x+1)}}$$
$$= \frac{2x+1}{x}$$

Salvatore

$$\frac{2x+2}{x+1} + \frac{1}{x} = \frac{2\cancel{(x+1)}}{\cancel{(x+1)}} + \frac{1}{x}$$
$$= 2 + \frac{1}{x}$$
$$= \frac{2(x)}{(x)} + \frac{1}{x}$$
$$= \frac{2x+1}{x}$$

Explain the difference in the methods used.

5. Randy says the only restriction of the variable x in Marissa and Salvatore's problem is 0. Cynthia says $x \neq 0, -1$. Who is correct? Explain your reasoning.

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6. Calculate each sum and difference. Make sure to list the restrictions for the variable and simplify the answers when possible.

a. $\frac{5x - 6}{x^2 - 9} - \frac{4}{x - 3}$

b. $\frac{x - 7}{x^2 - 3x + 2} + \frac{4}{x^2 - 7x + 10}$

c. $\frac{2x - 5}{x} - \frac{4}{5x} - 4$

d. $\frac{3x - 5}{4x^2 + 12x + 9} + \frac{4}{2x + 3} - \frac{2x}{3}$

PROBLEM 4 Can You Spot the Difference?

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Consider the worked example.

Determine the difference.

Example 1	Example 2
$\frac{3}{(x-1)} - \frac{3}{(x+1)}$	$\frac{1}{(x-1)} - \frac{1}{(x+1)}$
$= \frac{3(x+1)}{(x-1)(x+1)} - \frac{3(x-1)}{(x+1)(x-1)}$	$= \frac{1(x+1)}{(x-1)(x+1)} - \frac{1(x-1)}{(x+1)(x-1)}$
$= \frac{3x+3-3x+3}{(x-1)(x+1)}$	$= \frac{x+1-x+1}{(x-1)(x+1)}$
$= \frac{6}{(x-1)(x+1)}$	$= \frac{2}{(x-1)(x+1)}$

1. Describe the similarities and the differences in the structure of each example.

2. Consider the given expression and the resulting difference. What pattern do you notice?

3. If the numerators in Example 1 of the worked example were doubled, what would be the new answer?

4. Can you use this pattern to determine $\frac{4}{(x-2)} - \frac{4}{(x+2)}$? Explain your reasoning.

5. How would the pattern in the worked example change if you added the terms together?

Be prepared to share your solutions and methods.